JOURNAL OF COMPUTATIONAL PHYSICS **142,** 264–268 (1998)
ARTICLE NO. CP985945

NOTE

Reproducing the Flow Response to Actuator Motion

INTRODUCTION

Linear theory is used extensively in the varied mechanics disciplines, in particular for the analysis and control of flow instabilities. Simulations of the growth of small disturbances and of the reaction of the flow to active control systems typically require repeated runs of computationally intensive codes which model the flow. Here we present an efficient technique to accurately reproduce the linear flow responses to actuator motions in an unstable, laminar boundary layer on a flat plate which are usually produced by a Navier–Stokes code. As an application, we reproduce the disturbances generated by a wall-deflected membrane actuator attached to the plate. The flow responses are determined by a multigrid Navier–Stokes code developed by Liu, Liu, and McCormick [5] which has been modified to accept the inhomogeneous boundary conditions caused by the actuator movement [4]. The code is suited to perform direct numerical simulations (DNS) of transition in a three-dimensional boundary layer and has been used to develop design criteria for the actuators employed in our flow control method. Numerical simulations and experimental tests of the active laminar flow control method have demonstrated that our "smart wall" is capable of attenuating instability waves and delaying transition to turbulence [2, 3]. The ensuing brief re-acquaints the audience with the Duhamel superposition integral (DSI) and demonstrates the effectiveness of our technique which blends analytical theory with modern computational power.

METHOD

The transient flow response to actuator motion is time-dependent. Jean-Marie-Constant Duhamel (1797–1872) showed with his superposition integral that a linear problem of a time-dependent disturbance could be reduced to that of a single stepwise disturbance (e.g., [1]). Given the elementary response $u_o(\mathbf{x}, t)$ of the system to a unit stepwise forcing, DSI generates the response $u(\mathbf{x}, t)$ to a prescribed time-dependent forcing function $F(t)$ by evaluating the integral

$$u(\mathbf{x}, t) = F(0)u_o(\mathbf{x}, t) + \int_0^t u_o(\mathbf{x}, t - s)\frac{dF(s)}{ds}\, ds. \tag{1}$$

0021-9991/98 $25.00

In a time domain with constant time step this integral can be discretized as

$$u(\mathbf{x}, t) = F(0)u_o(\mathbf{x}, t) + \sum_{k=1}^{n} u_o(\mathbf{x}, t - \tau_k)[F(\tau_k) - F(\tau_{k-1})], \qquad (2)$$

where τ_k represents discrete time steps with $t \geq \tau_k$. Equation (2) demonstrates that the simulated solution is generated using a summation of the original step function solution in time, weighted by the forcing function. Since the forcing function is not limited by the DSI, any simulated response can be reproduced from a single original step function response. For example, we have composed the flow response to varied actuator motions in our numerical simulations of the smart wall using the flow response from a single DNS run and the DSI [4].

RESULTS

In the original formulation, the DSI uses an actuator in step function excitation to generate the building block flow response to actuator motion. However, as step function motion of an actuator is infeasible in the DNS, the actuator is linearly ramped up over two time steps to the maximum deflection and thereafter remains deflected. In the DNS, the time step and grid point density are related to the wave cycle and wave length of the most unstable mode generated by actuation. The flow responses have been shown to be independent of the time step and the grid point density [4], and they are in agreement with additional numerical and experimental techniques [3]. Since the wave cycle is 250 time steps per wave period, linearly ramping the actuator upward over two time steps is a good approximation to step function motion. Using a ramp function instead of a step function requires only a minor modification of the summation equation (2), but results in a more accurate simulation of the flow response with the DSI (similar to using trapezoids instead of rectangles in numerical integration).

The following flow responses are portrayed as the instantaneous streamwise disturbance velocity after two cycles of sinusoidal actuator motion in the neighborhood of the actuator in the streamwise/spanwise plane at $0.75\delta^*_{act}$. The actuator is located on the flow field centerline with parameters listed in Table 1. Motion of a membrane actuator in Blasius flow would normally generate streamwise and normal velocity disturbances, however, this actuator is restricted to generate only streamwise velocity disturbances. Figure 1 portrays the flow response to ramped actuator motion from the DNS, $u_o(\mathbf{x}, t)$. Figure 2 shows the flow response to sinusoidal actuator motion from the DNS. Figure 3 portrays a simulation

TABLE 1
Actuator and Flow Field Parameters

Parameter	Symbol	Value
Freestream velocity	U_∞	12.7 m/s
Kinematic Viscosity	ν	1.5e-5 m^2/s
Actuator		
Frequency for sinusoidal motion	f	73 Hz
Streamwise length	L	0.025 m
Spanwise width	W	0.125 m
Maximum deflection	D	0.020 mm
Location of center from leading edge	x_{act}	1.720 m
Boundary layer displacement thickness at actuator	δ^*_{act}	2.450 mm

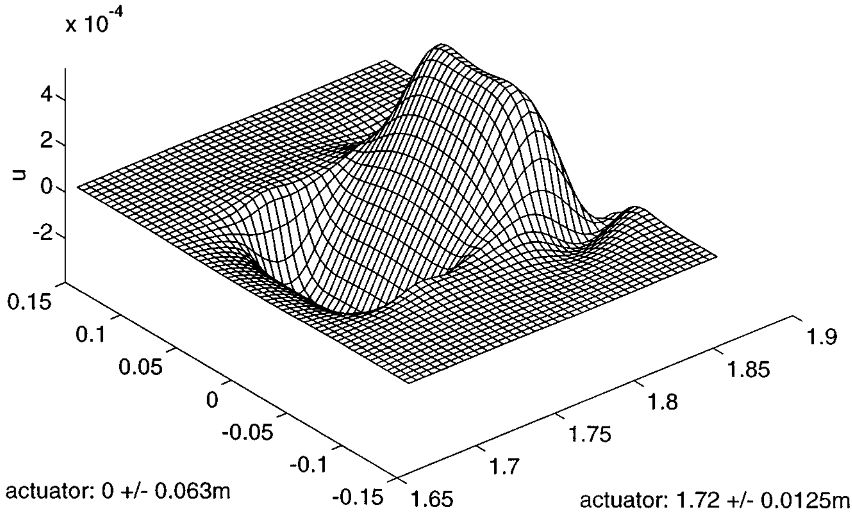

FIG. 1. Flow response to ramped actuator motion from DNS.

of the flow response to the same sinusoidal actuator motion, $u(\mathbf{x}, t)$. Figure 3 is generated using the DSI to superpose sinusoidal motion, $F(t) = \sin(t)$, on the flow response to ramped actuator motion, $u_o(\mathbf{x}, t)$. Figure 4 illustrates a centerline view of the flow response from the DNS and the simulation of the flow response using the DSI, which demonstrates the accuracy of this technique. Other flow field disturbances can be similarly superposed by prescribing the respective forcing function, $F(t)$.

DISCUSSION

We demonstrate that the linear flow disturbances generated by varied actuator motions can be reproduced using a single DNS run and the DSI. As this DNS code is time intensive,

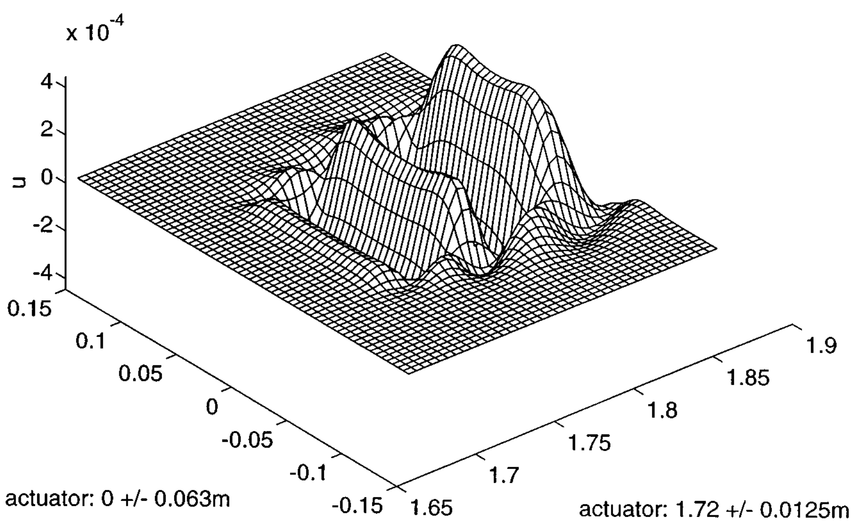

FIG. 2. Flow response to sinusoidal actuator motion from DNS.

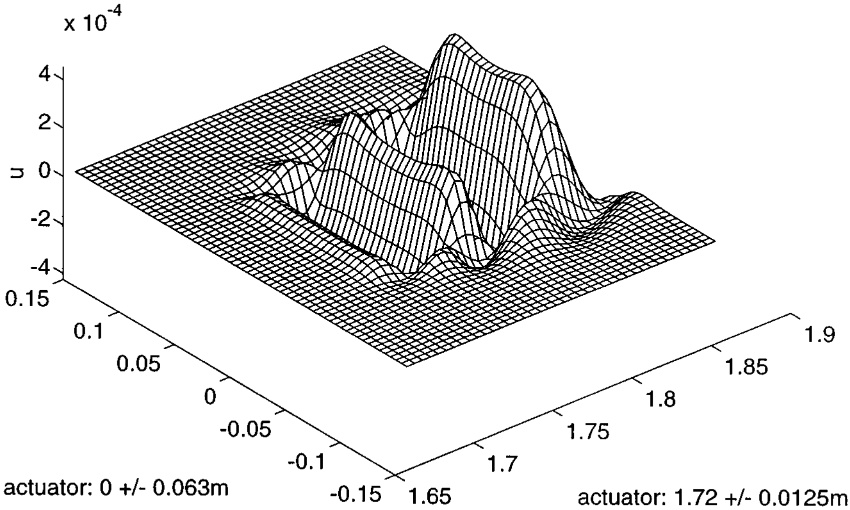

FIG. 3. Simulation of the flow response to sinusoidal actuator motion from DSI.

the DSI is used to quickly reproduce the flow response to actuation without the need to rerun the code. The technique is especially attractive in active control systems, where the solution is only needed at certain sensor locations. In summary, numerical simulations which portray linear solutions to varied mechanics problems could benefit by using the Duhamel

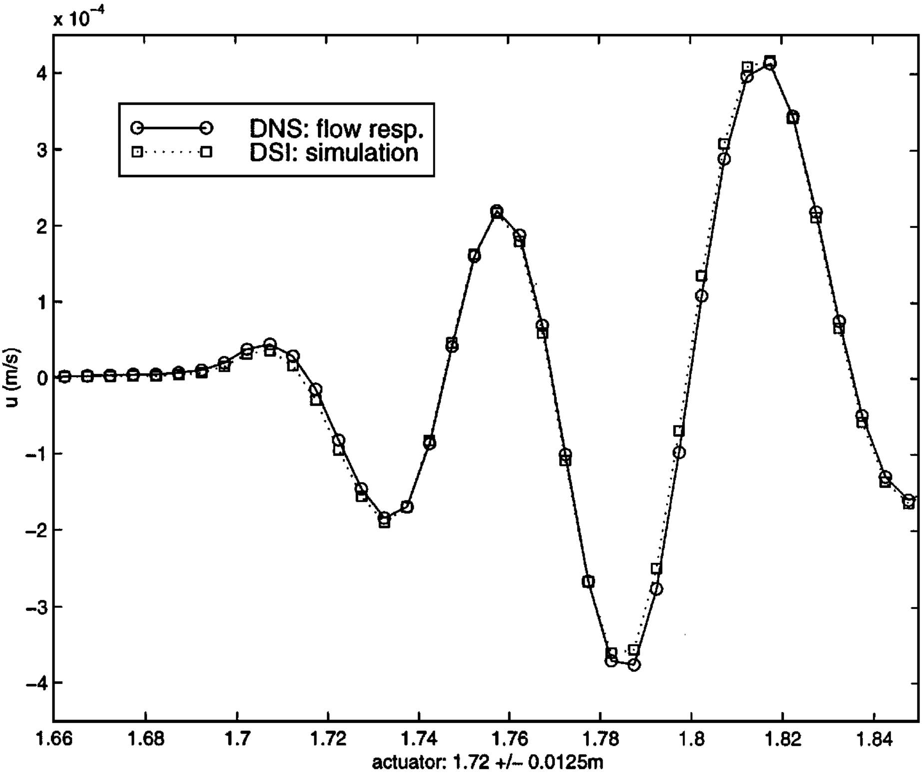

FIG. 4. Flow field centerline comparison of the flow response from DNS and the simulation of the flow response from DSI.

superposition integral to accurately and efficiently reproduce the solutions without the need to use the primary numerical solver.

ACKNOWLEDGMENTS

We thank Dr. James McMicheal and AFOSR Grants F49620-92-J-0339 and F49620-93-1-0135 which have supported this research.

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Received January 31, 1997; Revised February 3, 1998

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